injected in the indicated SCs of CHEs with high fluid flow rates G' and it was injected tangentially and with high velocity, increasing the flow rate of the liquid resulted in an increase of the spin rate of the fluid layer and the corresponding component of the total hydraulic resistance.

The observed linear relation between the dimensionless hydraulic resistance and the indicated complex is described approximately by the equation $\Delta P = 1 + 5kG'/G''$, which has the form (18). This once again confirms the correctness of the approach expounded here, though the last equation contradicts the relations proposed in [5, p. 83].

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CAPILLARY INSTABILITY OF AN EXTENDING JET

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The development and breakup of a capillary liquid jet, formed by axisymmetric extension, between two surfaces of a volume of liquid was investigated experimentally. The extension jet is formed during the operation of a monodispersed-drop generator (MDG) of the type "vibrating needle." The volume of the liquid participating in the extension process falls in the range $V_0 = (0.5\text{-}15.0)\cdot 10^{-11}$ m^3. The characteristics of the process of generation of extension jets are established. It is shown that instability of a cylindrical extension jet can arise both with and without axisymmetric oscillations.

1. Flows which can be termed extension jets arise in different processes (for example, deformation and fragmentation of drops in a gas flow, rupture of a connecting neck between drops in the process of merging of particles), resulting in axisymmetric extension of a liquid neck, i.e., a volume of liquid between two surfaces. In the study of such phenomena two aspects are distinguished: the form which the neck assumes under the action of the extension forces and surface tension and the breakup of a capillary extension jet.

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Consider a liquid cylinder which has an arbitrary length and initial radius a_0 and is subjected to axisymmetric extension with velocity u. In the cylindrical coordinate system (r, φ, z), where the z axis is the symmetry axis of the cylinder, the components of the velocity u are given by $u_r = -(1/2)Gr$, $u_\varphi = 0$, $u = Gz$ (G is a constant).

It can be shown [1, 2] that under conditions of axisymmetric extension the radius of the cylinder decreases with time as

$$a = a_0 \exp(-1/2Gt). \tag{1.1}$$

In the case of the extension of a liquid neck, however, it should be kept in mind that the form of the neck is more complicated than cylindrical. As is well known, the liquid between two solid coaxial circular plates with the same radius a and separation 2h assumes an axisymmetric form with constant average curvature of the surface. In addition, the stability and form of such a neck are determined only by the volume of the liquid V_0 and the ratio h/a [3]. The energetically most favorable (for a given V_0) shape is a shape with zero average surface curvature – a catenary surface. A liquid film stretched over a ring-shaped coaxial base forms such a surface [4]. The catenoid shape is stable if

$$0{,}47 \leqslant h/a \leqslant 0{,}67; \tag{1.2}$$

the right-hand side of the inequalities (1.2) is the condition for the existence of a catenary surface [3]. Obviously, as the neck stretches at some point h/a will exceed 0.67, i.e., the shape of the neck becomes unstable.

As is well known, a stationary cylinder of radius a becomes unstable when axisymmetric oscillations with wavelength λ, for which $x = 2\pi a/\lambda < 1$ (x is a dimensionless wave number), develop. Exponential growth of the amplitude of such oscillations results in fragmentation of the cylinder [5].

In [1, 2] it is shown that an extension jet also breaks up under the action of axisymmetric oscillations and it was found that the instability in this case exhibits some peculiarities. Thus in [2] it is concluded that a section where the relative amplitude increases exists only for oscillations for which the value of the initial wave number x_0 exceeds a critical value x_0^*, determined by the physical parameters of the system.

2. Capillary extension jets with Re ≫ 1 were investigated experimentally with the help of the MDG [6]. In the operation of the MDG (Fig. 1) the tip of the needle 1, secured on a flexible plate 2, is periodically inserted into a liquid, filling a capillary 3. The plate is vibrated by an electromagnet 4, which is powered by a low-frequency generator. The jet is extruded from the liquid when the needle is extracted from the capillary.

Such an object is conveniently studied by using the method of visualization of fast processes [7]. In this method the jet is illuminated with a pulsed source, synchronized with the generator and drives the oscillations of the plate with the needle. A Π-shaped pulse is required in order to trigger the flashlamp. Connecting the lamp through the Π-pulse generator, which has a pulse-delay unit, makes it possible to observe directly for as long as desired any phase of the process with an arbitrarily small step

In the present work we employed a Π-pulse generator which made it possible to delay the pulse by 10^{-6}-10^{-1} sec. This made it possible to measure with high accuracy both the geometric parameters of the jet (length ℓ of the jet, the diameter d_n of the jet in different sections, and the radius R of the drops formed) and the characteristic times t_n of different processes observed as the jet develops. In addition, the change in the geometric shape of the jet was recorded.

We studied different extension jets (necks), formed by extending the initial volume of the liquid in the range $V_0 = (0.5\text{-}15.0)\cdot 10^{-11}$ m³ with extension rate u = 0.6-2.0 m/sec. The experiments were performed with distilled water under laboratory conditions.

3. The development of the neck-jet was investigated from the time t = 0, when the needle stretched a volume V_0 of the liquid with no replenishment of the capillary. Next, the length of the neck ℓ, the diameter d of the neck in the central section, and the diameter d* of the capillary, i.e., the base diameter, were measured after a time interval $\Delta t = 2\cdot 10^{-5}$ sec.

The observations showed that the entire process of development of the neck-jet can be divided into several stages (Fig. 2). Up to some time t_1 the neck assumes a shape close to a

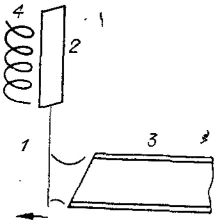

Fig. 1

catenary surface (Fig. 2a). Hence the extension velocities u are such that the thermodynamically most favorable relation is established between the volume V_0 of the liquid and the ratio ℓ/d^*. The ratio ℓ/d^* measured at time t_1 ($\ell/d = 0.83 \pm 0.3$) agrees well with the condition (1.2) of existence of a catenary surface.

Under further extension conical formations are separated at the needle and capillary and the central part of the neck-jet reforms into a cylinder (Fig. 2b). The shape of the cones does not change much, and the central cylindrical part is extruded. At the time t_0 necks form on both ends of the cylinder. The central cylindrical part transforms into a spindle shape (Fig. 2c). Over the time ~10^{-5} sec the diameter of the necks decreases to zero. The necks do not always rupture simultaneously. The first and second ruptures can be separated by time interval of ~10^{-5} sec. We note that the first rupture occurs at the time t_3. After the necks rupture a drop forms from the mass of liquid separated.

The measurements showed that the drop volume V_d is equal to the volume V_1 of the cylindrical part of the jet measured at any time between t_1 and t_2:

$$V_d = V_1 \pm 0{,}1V_1, \tag{3.1}$$

i.e., the exchange of liquid between the cones and the central part during the extension process is very insignificant.

The ratio of the length ℓ_1 of the cylinder to the diameter d in the central section of the cylinder, at time t_2 is independent of the extension rate u: $\ell_1/d = 5.5 \pm 2.5$. On the contrary, ratio of the length of the neck-jet ℓ to the diameter d, measured at time t_3 depends linearly on the extension rate u as $\ell/d = (A + Bu) \pm 2.5$ ($A = 4.2$ and $B = 0.6$ m/sec) and reaches a maximum value of 18 at $u \approx 2.0$ m/sec. The value of the constant A is determined by the conditions under which instability appears and the value of the constant B is determined by conservation of the volume of the extended liquid.

4. It was found experimentally that, in accordance with Eq. (1.1), as the neck-jet stretches and its diameter decreases, radial perturbations appear. Measurements of the geometric parameters of the neck-jet were performed starting at a fixed time, when the needle extended a column of liquid of volume V_0. The latter measurement was performed at time t_3, corresponding to the first rupture of the neck-jet. Figure 3 shows the characteristic form of the time dependenc eof the neck diameter d(t). The curve was constructed for a neck with parameters $V_0 = 2.7 \cdot 10^{-11}$ m^3 and $u = 2.1$ m/sec. It is interesting that the finite extension time t_3 is linearly related with the time t_4 at which the first maximum appears:

$$t_3 = 2t_4. \tag{4.1}$$

The first monotonically decreasing section of the time dependence d(t) corresponds to the catenoid stage. Thus the observed oscillations are disturbances which develop on the neck accompanying the formation and breakup of the central cylindrical part.

It is well known that the period T of the characteristic oscillations of different objects (jets [5], necks, drops with a bounded contact area [8], and drops [9]) is proportional

Fig. 2

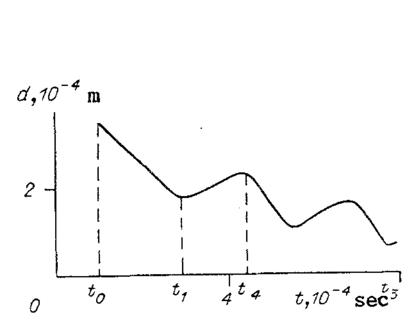

Fig. 3

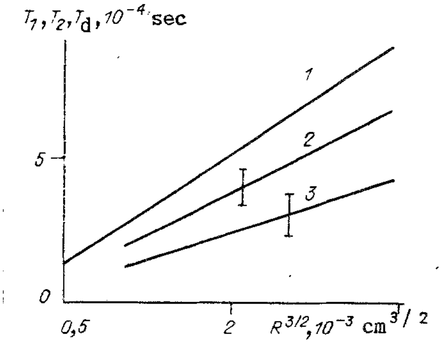

Fig. 4

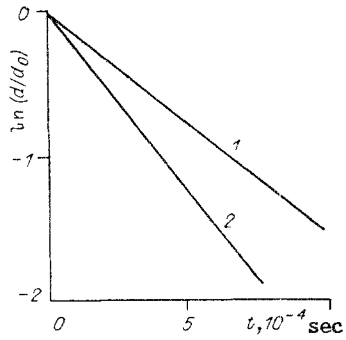

Fig. 5

to a combination of physical parameters of the system:

$$T \sim (\rho S^3/\sigma)^{1/2}. \quad (4.2)$$

Here σ and ρ are the surface tension and density of the liquid and S is the characteristic linear size. In the present case, since the volume V_1 of the central part of the jet is equal to the volume V_d of the drop (3.1) we choose the drop radius R as the characteristic size in the relation (4.2).

We now estimate how well the oscillations of the extension jet satisfy the relation (4.2). Over the extension time two oscillations with periods T_1 and T_2 (with $T_1 > T_2$), corresponding to the first and second maxima of the curve in Fig. 3, were observed on the jet.

Figure 4 shows the periods T_1, T_2, and T_d [the period of the oscillations of the drop formed on fragmentation of the neck ($T_d = 2.2(\rho R^3/\sigma)^{1/2}$, R is the drop radius] as functions of $R^{3/2}$. The lines 1, 2, and 3 determine T_d, T_1, and T_2. One can see that the periods T_1 and T_2 are proportional to $R^{3/2}$, i.e., they satisfy the law (4.2): $T_1 \sim T_2 \sim R^{3/2}$. In the course of the experiments, however, there were cases when the characteristic oscillations of the jet could not be separated, and the diameter of the neck-jets decreased exponentially according to Eq. (1.1) without any disturbances.

Figure 5 shows $\ln(d/d_0)$ as a function of time t for such jets with different extension velocities (for the lines 1 and 2 u = 0.77 and 1.45 m/sec) and the same initial volume $V_0 = 4.6 \cdot 10^{-11}$ m³ (d_0 is the diameter of the neck at the initial time t_0).

The existence of such cases confirms the result of [2] that there exists a critical value x_0^*. Suppose that when a jet is formed a random spectrum of initial oscillations forms on it. If this spectrum does not have any oscillations with wave number x_0, for which $x_0 > x_0^*$, then axisymmetric oscillations cannot develop, i.e., the increase in the relative amplitudes of any oscillations is completely compensated by a decrease in the absolute amplitudes as in Eq. (1.1) accompanying extension.

As experiments show, the initial spectrum of the oscillations is random. For the same values of the parameters V_0 and u an extension jet can form by either method. However the volume V_d of the separated mass of liquid in the case of extension of a jet without oscillations (index 1) is always less than in the case of breakup of jets with oscillations (index 2):

$$(V_d/V_0)_1 < 0{,}3 < (V_d/V_0)_2.$$

The experiments performed made it possible to determine the characteristics of the development and breakup of necks-jets under axisymmetric extension.

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SHOCK AND EXPANSION WAVES IN TRANSONIC FLOW

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The present article is concerned with the propagation of a shock wave and a simple expansion wave in transonic flow. Approximate relations are obtained for the flow parameters, and the resulting asymptotic dependences are analyzed as the small parameters of transonic theory tend to zero. The derived equaitons are used to show that a universal relation of the kind that exists in the linear theory of supersonic flow between the optimum permeability coefficient and the freestream Mach number M does not exist independently of the flow-obstructing body at $M \gtrsim 1$ for the Darcy condition customarily used in the theory of linear induction of pipe walls.

Nikol'skii [1] has succeeded in obtaining a universal relation for the optimum permeability coefficient of a perforated wall in the case of supersonic pipe flow (the influence of the wall on the flow in the pipe is assumed to be completely eliminated), satisfying the Darcy condition $v/u + R = 0$ (u and v are the horizontal and vertical components of the perturbed velocity, and R is the perforation ratio). Assuming small deviations of the velocity from the freestream velocity, Nikol'skii showed that the relation $v/u = -\sqrt{M_1^2 - 1}$ holds in an unbounded flow both in the shock wave and in the expansion wave generated by an obstructing body and does not depend on the parameters of the body (M_1 is the freestream Mach number of the supersonic flow). This relation is proposed in [1] as the condition for obtaining noninductive flow in a supersonic pipe, where it is required that the permeability coefficient of the walls satisfy the equation $R_{opt} = \sqrt{M_1^2 - 1}$.

Here we investigate the flow properties in a shock wave and in an expansion wave when $M \approx 1$. We show that a unique functional relation for the optimum permeability coefficient of the wall no longer exists for near-sonic supersonic flow, and instead it varies along the length of the pipe wall in each flow situation and differs for each experiment.

1. We consider the exact equations for an oblique compression shock [2, 3] (Fig. 1)

$$\frac{p_2}{p_1} - 1 = \frac{2\gamma}{\gamma+1}\left(M_1^2 \sin^2\beta - 1\right); \quad (1.1)$$

$$\operatorname{tg}\theta = \frac{\sin^2\beta - 1/M_1^2}{(\gamma+1)/2 - \sin^2\beta + 1/M_1^2}\operatorname{ctg}\beta; \quad (1.2)$$

$$M_2^2 \sin^2(\beta - \theta) = \frac{1 + [(\gamma-1)/2]\, M_1^2 \sin^2\beta}{\gamma M_1^2 \sin^2\beta - (\gamma-1)/2}. \quad (1.3)$$

Here p is the pressure, θ and β are the flow turning angle and the angle of inclination of the shock front, both measured relative to the x axis, β_1 is the supplementary angle measured from the y axis, the subscripts 1 and 2 refer to the state of the flow before and after the shock front, and γ is the adiabatic exponent.

Transonic flow is known to be characterized by two small parameters, whose interrelationship is dictated by the particular transonic regime characterized by the similarity parameter. The parameters $M_1^2 - 1$ and $M_2^2 - 1$ are convenient choices for the analysis of flow around a compression shock. The angles $\theta \ll 1$ and $\beta_1 \ll 1$ are also small in this case and can

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